

SCORE: ____ / 35 POINTS

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Let $R = \begin{bmatrix} -2 & 1 & 0 \\ 3 & -1 & 2 \end{bmatrix}$ and $S = \begin{bmatrix} 0 & -1 \\ 1 & 2 \\ -2 & 1 \\ 3 & 0 \end{bmatrix}$. Which product, RS or SR , exists? Find its value.

2×3 4×2

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$$SR = \begin{bmatrix} 0 & -1 \\ 1 & 2 \\ -2 & 1 \\ 3 & 0 \end{bmatrix} \begin{bmatrix} -2 & 1 & 0 \\ 3 & -1 & 2 \end{bmatrix} = \begin{bmatrix} -3 & 1 & -2 \\ 4 & -1 & 4 \\ 7 & -3 & 2 \\ -6 & 3 & 0 \end{bmatrix}$$

4×3

Let $G = \begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}$ and $H = \begin{bmatrix} 5 & 0 \\ -4 & 1 \end{bmatrix}$.

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[a] If $3H - 2X = G$, find X .

$$X = \frac{1}{2}(3H - G) = \frac{1}{2} \left(\begin{bmatrix} 15 & 0 \\ -12 & 3 \end{bmatrix} - \begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix} \right) = \frac{1}{2} \begin{bmatrix} 13 & 1 \\ -13 & 0 \end{bmatrix} = \begin{bmatrix} \frac{13}{2} & \frac{1}{2} \\ -\frac{13}{2} & 0 \end{bmatrix}$$

[b] Find G^2 .

$$GG = \begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 3 & -5 \\ 5 & 8 \end{bmatrix}$$

Solve the system $\begin{cases} x^2 + 2y = 5 \\ 2x - y = 8 \end{cases}$

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$$\begin{aligned} y &= 2x - 8 \\ x^2 + 2(2x - 8) &= 5 \\ x^2 + 4x - 16 &= 5 \\ x^2 + 4x - 21 &= 0 \\ (x + 7)(x - 3) &= 0 \rightarrow x = -7, 3 \end{aligned}$$

$$\begin{aligned} x = -7 &\rightarrow y = -22 & (-7, -22) \\ x = 3 &\rightarrow y = -2 & (3, -2) \end{aligned}$$

If matrix A has 3 rows, matrix B has 2 columns, matrix C has 4 rows, and $B = CA$, find the order of each matrix.

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$$\begin{matrix} B & = & C & A \\ 4 \times 2 & & 4 \times 3 & 3 \times 2 \end{matrix}$$

Find $\begin{vmatrix} 1 & -2 & 0 & 1 & -3 \\ -5 & 4 & 2 & -7 & 6 \\ -2 & 1 & 0 & 2 & -1 \\ 3 & 0 & 0 & -1 & 0 \\ -2 & -1 & 0 & 3 & 1 \end{vmatrix} = -2 \begin{vmatrix} 1 & -2 & 1 & -3 \\ -2 & 1 & 2 & -1 \\ 3 & 0 & -1 & 0 \\ -2 & -1 & 3 & 1 \end{vmatrix}$

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$$= -2 \left(3 \begin{vmatrix} -2 & 1 & -3 \\ 1 & 2 & -1 \\ -1 & 3 & 1 \end{vmatrix} + (-1) \begin{vmatrix} 1 & -2 & -3 \\ -2 & 1 & -1 \\ -2 & -1 & 1 \end{vmatrix} \right)$$

$$= -2 (3(-4 + 1 - 9 - 1 - 6 - 6) - (1 - 4 - 6 - 4 - 1 - 6))$$

$$= -2 (3(-25) - (-20))$$

$$= -2(-55)$$

$$= 110$$

Using Gauss-Jordan elimination, find the solution of the system $3x - 8y + 5z = 10$
 $2x - 3y + z = 2$
 $-x + 3y - 2z = -4$

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$$\begin{bmatrix} 3 & -8 & 5 & 10 \\ 2 & -3 & 1 & 2 \\ -1 & 3 & -2 & -4 \end{bmatrix} R_1 \leftrightarrow R_3$$

$$\begin{bmatrix} 1 & -3 & 2 & 4 \\ 0 & 3 & -3 & -6 \\ 0 & 1 & -1 & -2 \end{bmatrix} \frac{1}{3}R_2$$

$$\begin{bmatrix} 1 & 0 & -1 & -2 \\ 0 & 1 & -1 & -2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 3 & -2 & -4 \\ 2 & -3 & 1 & 2 \\ 3 & -8 & 5 & 10 \end{bmatrix} -R_1$$

$$\begin{bmatrix} 1 & -3 & 2 & 4 \\ 0 & 1 & -1 & -2 \\ 0 & 1 & -1 & -2 \end{bmatrix} R_3 + (-1)R_2$$

$$\begin{aligned} x - z &= -2 & x &= z - 2 \\ y - z &= -2 & y &= z - 2 \end{aligned}$$

$$\begin{bmatrix} 1 & -3 & 2 & 4 \\ 2 & -3 & 1 & 2 \\ 3 & -8 & 5 & 10 \end{bmatrix} \begin{matrix} R_2 + (-2)R_1 \\ R_3 + (-3)R_1 \end{matrix}$$

$$\begin{bmatrix} 1 & -3 & 2 & 4 \\ 0 & 1 & -1 & -2 \\ 0 & 0 & 0 & 0 \end{bmatrix} R_1 + 3R_2$$

$$\begin{aligned} x &= t - 2 \\ y &= t - 2 \\ z &= t \end{aligned}$$